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Adding D7-branes to the Polchinski-Strassler gravity background

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Abstract

We motivate and summarize our analysis of hep-th/0610276 in which we consider D7-brane probe embeddings in the Polchinski-Strassler background with $\mathcal{N} = 2$ supersymmetry. The corresponding dual gauge theory is given by the $\mathcal{N} = 2^*$ theory with fundamental matter. Based on a talk given by C. Sieg at the RTN Workshop 2006, Napoli.

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1 Introduction

The AdS/CFT correspondence [1] conjectures a duality between a string theory in a curved background with a boundary and a gauge theory that lives on this boundary. In a particular case, type IIB string theory in $\text{AdS}_5 \times S^5$ with N units of Ramond-Ramond 5-form flux is conjectured to be dual to the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory with gauge group $SU(N)$, living on the four-dimensional boundary of the string background. In terms of the 't Hooft coupling constant $\lambda = g_{\text{YM}}^2 N$, the gauge theory is strongly coupled when the string theory reduces to supergravity. The latter is seen from $\lambda = \frac{R^4}{\alpha'^2}$ which relates λ to the curvature radius R of $\text{AdS}_5 \times S^5$ and the squared string length α' . To obtain classical supergravity requires also that the $N \rightarrow \infty$ limit is taken with λ held fixed. This sends the string coupling constant g_s to zero, as follows with $g_{\text{YM}}^2 = 4\pi g_s$ from the definition of λ . The gauge theory becomes planar in this limit.

The $\mathcal{N} = 4$ SYM theory is not confining but superconformal, and moreover it only contains fields that transform in the adjoint representation of the gauge group. To make contact with the gauge theories that enter the standard model of particle physics, and in particular with the confining QCD, one therefore has to go beyond the above described setup. To find a theory that resembles QCD, we should break the conformal symmetry, (some of the) supersymmetries, and add (quark) fields that transform in the fundamental representation of the gauge group. The latter can be realized by adding D-brane probes [2]. The assumption that also in cases with less symmetries a gauge/gravity correspondence holds goes beyond the original conjecture. However, believing in its validity, we can continue using it to construct gravity duals to more realistic gauge theories.

A particularly interesting example for a deformed supergravity background which displays a rich physical structure at low energies is due to Polchinski and Strassler [3]. As far as the D7-brane probe embedding for the addition of fundamental matter is concerned, this background has the appealing feature that in the far UV, its dual field theory returns again to four-dimensional $\mathcal{N} = 4$ SYM. This allows for the identification of boundary field theory operators from the asymptotic behaviour of the embedding functions, in analogy to the standard AdS/CFT dictionary.

2 The Polchinski-Strassler background

The Polchinski-Strassler (PS) background [3] is obtained as the gravity dual to $\mathcal{N} = 4$ SYM theory, deformed by adding mass terms $\frac{1}{g_{\text{YM}}^2} m_p \text{tr} \Phi_p^2$, $p = 1, \dots, 3$ for the three adjoint chiral $\mathcal{N} = 1$ supermultiplets Φ_p , keeping massless only the $\mathcal{N} = 1$ gauge supermultiplet. The dimensionful mass parameters m_p clearly break conformal invariance. How much of the supersymmetry is broken depends on the concrete values of the m_p . Polchinski and Strassler [3] have discussed the case $m_1 = m_2 = m_3$. The corresponding gauge theory, in which one of the four supersymmetries is preserved, is known as the $\mathcal{N} = 1^*$ theory. Instead, if mass terms are added for only two of the three chiral mul-

triplets, e.g. $m_1 = 0$, $m_2 = m_3$, the theory preserves two supersymmetries and is called $\mathcal{N} = 2^*$ theory.

On the gravity side the modifications correspond to the presence of 3-form flux G_3 built with an imaginary anti-selfdual (IASD) 3-tensor T_3 , i.e. $(\star_6 + i)T_3 = 0$. The effect of G_3 on the gravity background is better understood if one considers the background generating stack of N D3-branes before the decoupling limit for the gauge/gravity correspondence is taken. The backreaction of G_3 on the D3-branes is an example of the dielectric effect found by Myers [4], i.e. it can be interpreted as a polarization of the D3-branes into their transverse directions to extended brane sources. The near horizon region, into which the decoupling limit zooms, is no longer given by $\text{AdS}_5 \times \text{S}^5$ – in fact, its full form is not known explicitly. However, it is asymptotically $\text{AdS}_5 \times \text{S}^5$. Therefore, at sufficiently large distance from the sources one can treat the extension of the sources perturbatively, such that the background can be computed as a perturbation series around $\text{AdS}_5 \times \text{S}^5$.

The background fields receive contributions from the backreaction of the non-vanishing 3-form flux G_3 on the gravity background. The linear or order $\mathcal{O}(m)$ contributions are discussed in the original paper [3], where it is shown that apart from the presence of the 2-form potentials \tilde{C}_2 and B of G_3 , also 6-form potentials are induced. At quadratic order $\mathcal{O}(m^2)$ the dilaton [3, 5] as well as the metric and 4-form potential C_4 are modified [5]. The quadratic order background has been completed in [6] with the 8-form potential C_8 . In terms of the potentials \tilde{C}_2 and B of G_3 we have found

$$C_8 = -\frac{1}{6} \left(e^{2\hat{\phi}} \tilde{C}_2 \wedge \tilde{C}_2 + B \wedge B \right) \wedge \hat{C}_4, \quad (2.1)$$

where $\hat{\phi}$ and \hat{C}_4 denote the uncorrected dilaton and 4-form potential, respectively. At cubic order $\mathcal{O}(m^3)$ it has been shown [7] that the flux G_3 itself is corrected in such a way that in the dual gauge theory a gaugino condensate is generated.

We have analyzed the symmetry structure of the $\mathcal{N} = 2$ PS background and found that the D3-branes in the deep interior are polarized such that they form two overlapping spheres,

$$(y^5)^2 + (y^6)^2 + (y^7)^2 = r_0^2, \quad (y^7)^2 + (y^8)^2 + (y^9)^2 = r_0^2, \quad (2.2)$$

where the y^i are the six directions perpendicular to the D3-branes (see also Table 1), and r_0 is a constant that depends on the polarizing potential B . This geometry preserves an $SU(2) \times SU(2) \simeq SO(4)$ symmetry very suitable for the embedding of a D7-brane. We see that there is no polarization in the y^4 direction. Let us remark that close to the extended D3-brane sources with extension r_0 the perturbation theory breaks down, and the background is not known.

3 D7-brane probes

The original $\mathcal{N} = 4$ SYM theory and hence also its mass deformations only contain fields that transform in the adjoint representation of the gauge group. Karch and Katz [2]

	x^μ	y^4	y^5	y^6	y^7	y^8	y^9
D3	+	−	−	−	−	−	−
D7	+	−	+	+	−	+	+

Table 1: Orientation of the background generating D3 branes and of the D7-brane probe in the 10-dimensional spacetime.

have proposed to add N_f field flavours which transform in the fundamental representation (henceforth denoted as quarks) by embedding N_f spacetime-filling D7-branes into $\text{AdS}_5 \times \text{S}^5$. In the brane picture, the N_f quark flavours correspond to open strings with one of their endpoints sitting on the stack of D3-brane and the other one ending on one of the N_f D7-branes. The choice $N_f \ll N$ thereby allows one to neglect the backreaction of the D7-branes on the background, i.e. to consider them as brane probes.

It turns out that a D7-brane probe spans an $\text{AdS}_5 \times \text{S}^3$ inside $\text{AdS}_5 \times \text{S}^5$. Along the holographic direction r of $\text{AdS}_5 \times \text{S}^5$ the brane fills all of AdS_5 from the boundary at $r \rightarrow \infty$ down to a minimal value $r = \hat{u}$ at which it terminates. In the AdS/CFT correspondence the radial coordinate r has the interpretation of an energy scale with small and large r corresponding to the IR and UV regimes in the dual gauge theory. Therefore, the termination of the D7-brane at $r < \hat{u}$ means that there exists an energy threshold below which the corresponding quark degree of freedom cannot be excited. The threshold is proportional to the quark mass m_q . The precise relation reads

$$m_q = \frac{R}{2\pi\alpha'} \hat{u} . \quad (3.1)$$

Adding a D7-brane probe with $m_q = 0$ clearly breaks the $SO(6)$ isometries of the S^5 to $SO(4) \times U(1)$. In the dual gauge theory this corresponds to a breaking of the $SU(4)$ R-symmetry to $SU(2) \times U(1)$. The corresponding gauge theory is $\mathcal{N} = 2$ SYM with massless fundamental matter, which is still a superconformal theory. In addition, for $m_q \neq 0$ the $U(1)$ factor, which rotates the directions transverse to the D7-brane probe, is broken. The dual gauge theory is $\mathcal{N} = 2$ SYM with fundamental matter with mass m_q . This theory has an $SU(2)$ R-symmetry. The fundamental mass breaks the conformal symmetry.

In [6] we have studied the embedding of D7-brane probes into the $\mathcal{N} = 2$ Polchinski Strassler background. We have chosen the $\mathcal{N} = 2$ case with masses $m_1 = 0$, $m_2 = m_3$ instead of the $\mathcal{N} = 1$ case, since this background preserves an $SO(4)$ symmetry in the transverse directions. An embedding of the D7-brane probe as shown in table 1 then allows the D7-brane embedding coordinates to depend on the radial variable ρ in the four parallel directions y^5, y^6, y^8, y^9 only. These directions are dual to the complex scalars of the massive chiral multiplets Φ_2, Φ_3 of the gauge theory. The two embedding directions y^4 and y^7 correspond to the complex scalar of the massless Φ_1 .

In order to be able to find analytic solutions for the embeddings, we have decomposed the embedding coordinates y^4, y^7 themselves into the constant unperturbed embedding

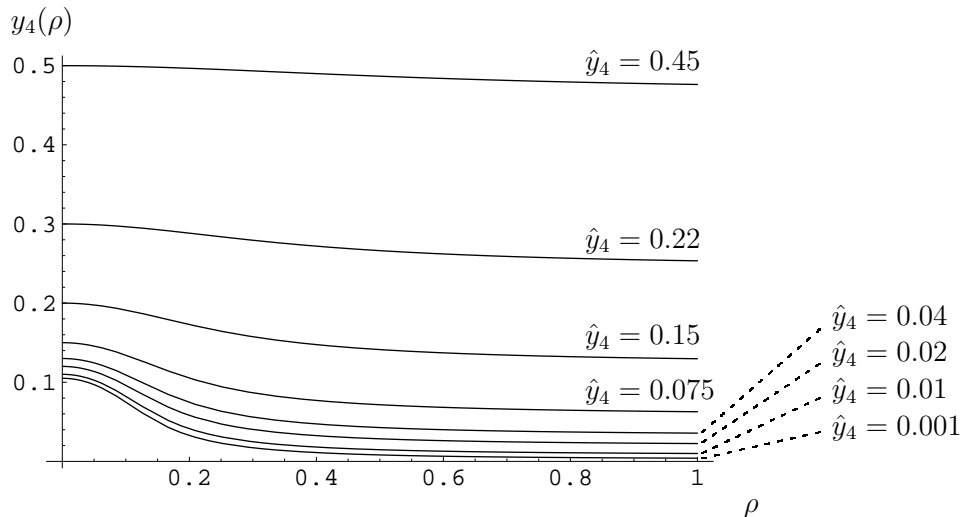


Figure 1: Embeddings along y_4 with different boundary values \hat{y}^4 . The background generating D3-branes are not polarized in this direction. The AdS radius has been set to $R = 1$ and the adjoint deformation to $m = 0.2$. Lengths are dimensionless and measured in units of R .

coordinates \hat{y}^m in $\text{AdS}_5 \times S^5$, and a non-constant correction \tilde{y}^m according to

$$y^m(\rho) = \hat{y}^m + \tilde{y}^m(\rho) , \quad m = 4, 7 . \quad (3.2)$$

The boundary conditions are fixed by $y^m(\rho \rightarrow \infty) = \hat{y}^m$ and by the requirement that the corresponding solution is regular at $\rho = 0$. The unique analytic solution is valid whenever $\hat{y}^m \gtrsim \tilde{y}^m$ which according to (3.1) requires a sufficiently large quark mass m_q .

The constant D7-brane probe embeddings \hat{y}^m in pure $\text{AdS}_5 \times S^5$ acquire corrections \tilde{y}^m not before the quadratic order $\mathcal{O}(m^2)$ in the G_3 -form flux perturbation, such that we were forced to work at least at this order. We therefore had to deal with a complicated backreaction that e.g. affects the metric such that it becomes non-diagonal [5]. We also completed the known corrections to the dilaton, metric and 4-form with the 8-form C_8 given in (2.1). It also is of order $\mathcal{O}(m^2)$, and it couples to the worldvolume of the D7-brane probes.

For the D7-brane probe oriented as shown in table 1 we have then computed the action as the sum of the Dirac-Born-Infeld and Chern-Simons action. From this result we extracted the equations of motion for the corrections \tilde{y}^m .

We have found that the embedding with $y^4 = y^7 = 0$ is not corrected up to order $\mathcal{O}(m^2)$. Even if this embedding enters the regime where the perturbative expansion of the background breaks down, we expect that it is preserved also in the full solution. Furthermore, the two embeddings with either $y^7 = 0$ or $y^4 = 0$ are singled out: the coordinate that is zero is not corrected up to order $\mathcal{O}(m^2)$. These embeddings with $y^m = (u, 0)$ and $y^m = (0, u)$ are perpendicular to or respectively contain this direction

into which the original background generating D3-branes are polarized.

Applying the method of the holographic renormalization [8, 9, 10] we have found that the regularized on-shell action can be made to vanish by choosing an appropriate scheme, i.e. by adding appropriately chosen finite counterterms. In particular, this demonstrates the absence of a quark condensate, which in a generalization of the standard AdS/CFT dictionary would follow from the ρ dependence of the embedding. The absence of the quark condensate is a necessary condition for supersymmetry to be preserved.

We have also studied the two different embeddings in y^4 and y^7 direction numerically. Our findings for the embeddings along y^4 with different boundary values \hat{y}^4 are shown in figure 1. We see that the embeddings are repelled from the singularity in this direction, and that for larger boundary values \hat{y}^4 and therefore according to (3.1) for larger quark masses m_q the embeddings approach the constant embedding in $\text{AdS}_5 \times S^5$. On the other hand, the embeddings in the y^7 direction feel the effect of the brane shell. Although our $\mathcal{O}(m^2)$ approximation breaks down at scales of the order of this expected brane shell, we see evidence in our numerical result that the D7 brane merges with this shell and thus is repelled much stronger from the origin than the brane embedded in the y^4 direction. We refer the reader to [6] for details about the y^7 embedding.

To obtain the meson mass M as a function of the quark mass m_q [11] we have numerically analyzed the fluctuations around the embedding along y^7 . In the regime where the expansion of the gravity background is valid it is of the form $M^2 = b m_q^2 + c$, where b and the mass gap c are some constants. This supergravity result agrees with field theory expectations: The presence of the adjoint masses m leads to a mass gap in the meson spectrum M . Moreover, the above functional form does not contain a dependence on the quark bilinear since this would appear as a linear dependence on m_q . Therefore, the meson spectrum found also supports our holographic renormalization result that the quark condensate vanishes and hence supersymmetry can be preserved.

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